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Electric-field-induced magnetoresistance of lateral superlattices

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Abstract. The transverse electric field E_y which appears in lateral superlattices (SLs) in the presence of a high applied electric field E_x and a low magnetic field $\boldsymbol{H}$ normal to the SL plane ($\boldsymbol{H}\|$OZ) is calculated. When the electron energy spectrum is non-additive, the field E_y contains both the Hall field and the spontaneous transverse electric field which exists without $\boldsymbol{H}$. The field E_y is a multiple-valued and sign-changing function of E_x. The kinetic potential whose minimum corresponds to the stationary state of the non-equilibrium electron gas is used. The magnetoresistance caused by the appearance of a spontaneous transverse EMF is investigated.

In the present work the peculiarities of the Hall effect and the magnetoresistance (MR) in lateral superlattices (SLs) in a non-quantizing low magnetic field and a high electric field are investigated. Such SLs are constructed on the basis of dimensionally quantum-confined layers of $Al_xGa_{1-x}As$. The energy of a miniband is given by [1, 2]

$$\epsilon(\boldsymbol{p}) = \Delta - \tfrac{1}{2}\Delta\left[\cos\left(\frac{p_1 d_0}{\hbar}\right) + \cos\left(\frac{p_2 d_0}{\hbar}\right)\right] \tag{1}$$

where 2Δ is the allowed miniband width, p_1 and p_2 are the Cartesian components of the carrier crystal momentum $\boldsymbol{p}$ and d_0 is the period of the SL. With respect to the spectrum (1) the peculiarities of the Hall effect and of the MR are connected closely with the choice of the direction of applied electric field relative to the principal axes of the SL; we assume that this direction makes an angle of 45° with some of these axes. Correspondingly we define the OX axis as directed along the applied field. In this reference frame the spectrum (1) becomes non-additive:

$$\epsilon(\boldsymbol{p}) = \Delta - \Delta\cos\left(\frac{p_x d}{\hbar}\right)\cos\left(\frac{p_y d}{\hbar}\right) \tag{2}$$

where $d = d_0 2^{1/2}$.

In [3] the conductivity in the case of a non-additive non-parabolic dispersion law of form (2) has been studied, the conductor being an open circuit in the transverse (with respect to longitudinal electric field E_x) direction. It was shown that in the absence of a magnetic field the spontaneous appearance of a transverse EMF is possible. This effect presents an example of a non-equilibrium second-order phase transition, in which the transverse EMF plays the role of order parameter and the applied field is the controlling parameter. The existence of the transverse EMF at $\boldsymbol{H} = 0$ must evidently influence the magnitude of

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the transverse field when the magnetic field ($\boldsymbol{H}\|\mathrm{OZ}$) is present. As far as the non-linear effects are considered, the 'pure' Hall effect cannot be separated in this case. Thus when speaking about the Hall field we mean the transverse electric field which includes both above-mentioned factors.

To calculate the current density $\boldsymbol{j}_0$ due to the carriers with the dispersion law of form (2) we confine ourselves to the semiclassical and single-band approximation $\Delta \gg \tau^{-1}\hbar$, eEd, and $eEd \ll \epsilon_g$, where ϵ_g is the forbidden miniband width and τ is the mean free time of electrons.

The magnetic field is assumed to be non-quantizing, i.e. $\hbar\omega_c \equiv |eH\Delta d^2/c\hbar| \ll T$ (T is the temperature in energy units), and weak, i.e. $\omega_c\tau \ll 1$.

If we change to dimensionless variables, i.e. $\boldsymbol{p}d/\hbar \to \boldsymbol{p}$; $\boldsymbol{E}e\tau d/\hbar \to \boldsymbol{E}$, $\omega_c\tau \to \omega_c$, and $t/\tau \to t$, the necessary charge motion equation may be presented as follows:

$$\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = \boldsymbol{E} + \boldsymbol{v}(\boldsymbol{p}) \times \boldsymbol{\omega}_c \qquad (\boldsymbol{\omega}_c\|\boldsymbol{H}) \tag{3}$$

where $\boldsymbol{v} = 1/\Delta(\partial\epsilon/\partial\boldsymbol{p})$ is the dimensionless charge velocity.

We use the Boltzmann kinetic equation with the τ = constant approximation of the collision integral. Then [4]

$$\boldsymbol{j} = \int_0^\infty \boldsymbol{v}(\boldsymbol{p}(t)) \exp(-t)\, \mathrm{d}t \tag{4}$$

where $\boldsymbol{j} = \boldsymbol{j}_0\hbar/en\Delta d$ is the dimensionless current density, $\boldsymbol{p}(t)$ is the solution of equation (3) with the initial condition $\boldsymbol{p}(0) = 0$, and n is the carrier concentration in the layer. Using the linear approximation on ω_c and (2)–(4) we get

$$j_x = j_x^{(0)} + \frac{\omega_c E_y}{E_x^2 - E_y^2}\left\{\frac{1 + 2(E_x^2 + E_y^2)}{(1 + 4E_x^2)(1 + 4E_y^2)} - \frac{j_y^{(0)}}{E_y}\right\} \tag{5}$$

$$j_x^{(0)} = \frac{E_x(1 + E_x^2 - E_y^2)}{(1 + E_x^2 + E_y^2)^2 - 4E_x^2E_y^2}. \tag{6}$$

The expressions for j_y and $j_y^{(0)}$ have the form (5) and (6) with the replacement $y \leftrightarrow x$ and $\omega_c \to -\omega_c$. Note that, as these formulae are linear approximately with respect to the magnetic field, they are exact with respect to the electric field.

For a given applied field E_x the transverse field E_y is determined by the boundary conditions. In the following we assume the sample to be an open circuit:

$$j_y = 0. \tag{7}$$

This condition represents the equation for a transverse field $E_y = E_y(E_x)$. Provided that $\omega_c = 0$, equation (7) has the following solutions:

$$E_y = 0 \tag{8}$$

$$E_y = \pm\sqrt{E_x^2 - 1} \qquad (|E_x| > 1). \tag{9}$$

As was shown in [3], for $|E_x| < 1$ the null solution (8) is stable with respect to fluctuations of the field E_y whereas, if $|E_x| > 1$, then the non-zero solution (9) applies. Thus at $|E_x| > 1$ the transverse field appears in one of two mutually opposite directions; the direction is selected by a random fluctuation or by an initial ('seed') inhomogeneity. In this case the above-mentioned non-equilibrium (kinetic) second-order phase transition takes place at the bifurcation point $|E_x| = 1$. The appearance of the transverse field (9) represents perhaps the simplest example of self-organization in the non-equilibrium quasi-two-dimensional electron gas.

Note also that the current–voltage characteristic is influenced by the non-zero transverse field in the following way:

$$j_x = \frac{E_x}{1+E_x^2} \qquad 0 \leqslant |E_x| \leqslant 1 \tag{10}$$

$$j_x = (2E_x)^{-1} \qquad |E_x| \geqslant 1. \tag{11}$$

In the absence of a transverse field the whole current–voltage characteristic would be described by equation (10).

For $\omega_c \neq 0$, equation (7) has been solved numerically. The results of calculations under the conditions $\omega_c = 0.1$ and $\omega_c = 0$ are presented in figure 1. The region $E_x < 1$ attracts attention because, on the one hand, in the absence of magnetic field we have $E_y = 0$ and, on the other hand, the Hall effect appears unusually (the field E_y has a maximum and changes sign). In spite of the fact that the weak magnetic field is considered ($\omega_c \ll 1$) it plays the principal role because it smears out the phase transition and enforces the system to make a completely determinate selection between the conditions which are of equal probability at $\omega_c = 0$. Thus forced bifurcation takes place.

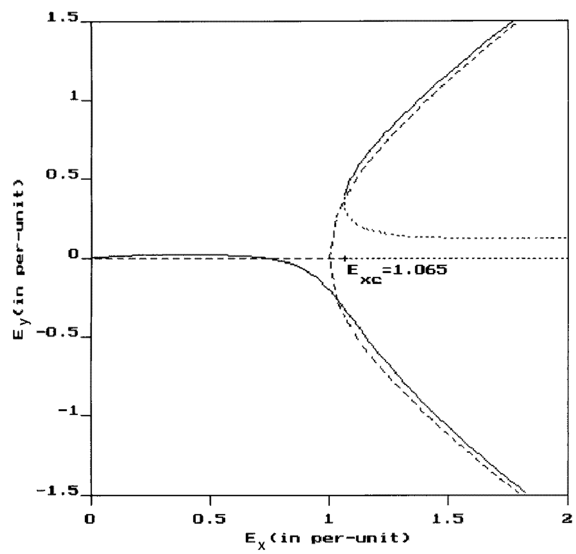

Figure 1. Dependence of the transverse field E_y on the applied field E_x: ——, stable state at $\omega_c = 0.1$; – – –, stable state at $\omega_c = 0$; · · · · · ·, unstable states.

When investigating the stability of obtained solutions we start from the condition [5]

$$\frac{\mathrm{d}j_y}{\mathrm{d}E_y} > 0 \qquad (E_x = \text{fixed}) \tag{12}$$

the fulfilment of which means that, near the stable stationary values of E_y defined by (7), the small fluctuation in the transverse field tends to zero asymptotically. The asymptotically stable states defined by criterion (12) are delineated in figure 1 by solid curves. Note that at $E_x > 1.065$ the Hall field has two stable values (bistability) for a given E_x. (The possibility of 'switching' of the Hall field together with the change in sign in the one-dimensional SL has been mentioned in [6].) In figure 2 the current–voltage characteristic is presented which has been found from (5) and (6) and by the values of E_y obtained at $\omega_c = 0.1$.

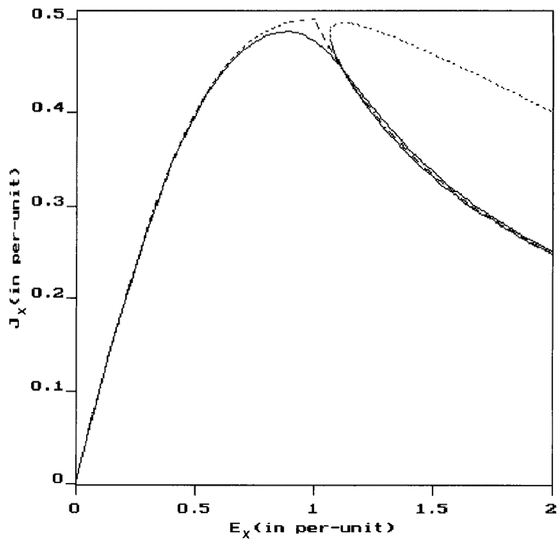

Figure 2. The current–voltage characteristic at $\omega_c = 0$ (– – –) and at $\omega_c = 0.1$ (——): $\cdots\cdots$, unstable states of E_y (see figure 1) at $\omega_c = 0.1$.

We also investigate the stability using the method proposed in [3] which consists of the following. The function

$$\Phi(E_y) = \int_0^{E_y} j_y(E'_y)\,\mathrm{d}E'_y + \text{constant} \qquad (E_x = \text{fixed}) \tag{13}$$

is defined. The conditions (7) and (12) may now be formulated in the form

$$\frac{\mathrm{d}\Phi}{\mathrm{d}E_y} = 0 \qquad \frac{\mathrm{d}^2\Phi}{\mathrm{d}E_y^2} > 0. \tag{14}$$

These formulae mean that in the given non-equilibrium situation the function (13) attains its minimum in the stationary state. Thus the function Φ may be regarded as the analogue of a thermodynamic potential for equilibrium systems. This analogy permits us in particular to use the conventional Landau method (used in the theory of equilibrium phase transitions) to investigate the stability of solutions of equation (7). This approach confirms the conclusions deduced using (7) and (12) and provides the possibility of distinguishing the absolute minimum from local minima. In particular it appears that the depth of the minimum of the 'potential' Φ on the lower stable branch (figure 1) is greater than the corresponding depth on the upper stable branch. When $E_{xc} = 1.065$, the condition $\mathrm{d}^2\Phi/\mathrm{d}E_y^2 = 0$ is fulfilled at the critical point $E_{yc} = 0.45$. Note that the dotted branch in figure 1 corresponds to the maximum of the 'potential' Φ.

The magnetoresistance is defined in a standard way according to

$$\frac{\Delta\rho}{\rho} = \frac{\rho(\omega_c) - \rho(0)}{\rho(0)} \tag{15}$$

where $\rho(\omega_c) = E_x/j_x$; in addition we find the MR as a function of E_x, the value of ω_c being fixed. Using the values of E_y obtained by (7) we get $\Delta\rho/\rho \sim \omega_c^2$. Therefore, generally speaking, the terms $\sim \omega_c^2$ must be considered. This procedure contains no principal difficulties but is rather cumbersome. Therefore we restrict ourselves by graphical illustration of the result at $\omega_c = 0.3$ (figure 3). Curve 1 corresponds to a lower stable branch, and curve 2 corresponds to an upper stable branch, these branches being analogous to those presented by solid curves in figure 1. It should be noted that in this case the MR presented in figure 3 differs insignificantly from that calculated from (5) and (6).

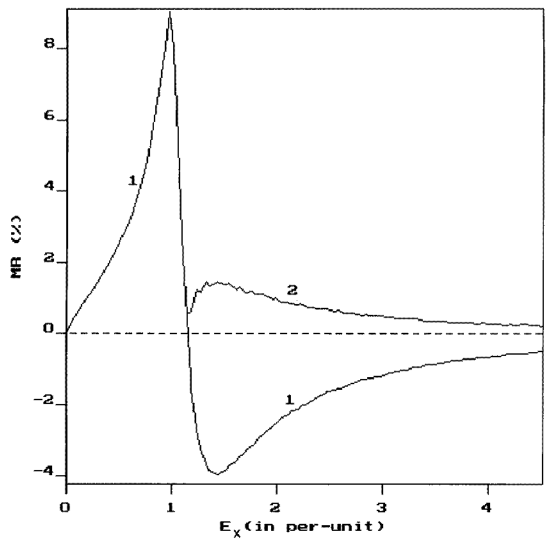

Figure 3. The MR as a function of applied field at $\omega_c = 0.3$: line 2, unstable states.

At $E_{xc} = 1.14$ the MR vanishes and at $E_x > E_{xc}$ it becomes negative. Such behaviour of the MR is caused by the appearance of a spontaneous transverse EMF. This conclusion may be confirmed by the fact that the MR is positive at any E_x in the case when the spontaneous EMF is not considered. The observed peculiarities of the transverse field and MR are related finally to the fact that the spectrum (2) is bounded, non-parabolic (or non-quadratic), anisotropic and mostly with the non-additivity of (2). Numerical estimations of the effects considered lead to estimations of the measurement units of electric intensity ($\hbar/e\tau d$) and current density ($en\Delta d/\hbar$). For $d = 10^{-6}$ cm, $\Delta = 10^{-2}$ eV, $n = 10^{15}$ cm^{-3}, $\tau = 10^{-12}$ s we get $\hbar/e\tau d \simeq 660$ V cm^{-1} and $en\Delta d/\hbar \simeq 2.4 \times 10^3$ A cm^{-2}. In this connection the condition $\omega_c\tau = 1$ corresponds to the magnetic intensity $H \simeq 400$ Oe.

In conclusion we shall discuss shortly the $\tau =$ constant approximation of the collision integral used in the present paper. Naturally this approximation does not take into account the influence of anisotropy and non-parabolicity of the spectrum on the dispersion of electrons, and for two-dimensional problems it is less than for one-dimensional problems [7]. At the same time the comparison of kinetic coefficients obtained in the $\tau =$ constant approximation for a one-dimensional SL shows satisfactory consistency with those calculated when the anisotropy is considered (in the limiting case of a low electric field) [8], the difference being only in factors of order 1. Also we have made a series of numerical calculations of the current density based on Chamber's [9] method in which the power dependence $\tau = \tau_0\epsilon^s$ has been used. The results show that the $\tau =$ constant approximation describes correctly the qualitative peculiarities of regarded phenomenon. We also believe that thorough analysis of numerous experiments on electron transport in a one-dimensional SL carried out in [10] indicates that in many situations the case $\tau =$ constant may prove to be quite realistic.

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